Supplementary Material

Supplement to "Multiple filtering devices for the estimation of cyclical DSGE models": Appendixes

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Appendix A: The NK model

The model we use is a version of a textbook new-Keynesian model (e.g., Gali (2008)) with a few exceptions. We assume habit in consumption, a preference shock, and, as in Smets and Wouters (2003, 2007), that the elasticity of variety of goods is an exogenous stochastic process.

A.1 *Households*

The representative household prefers to consume a variety of goods: the consumption basket is

$$C_t = \left(\int_0^1 C_t(j)^{(\epsilon_t - 1)/\epsilon_t}\, dj\right)^{\epsilon_t/(\epsilon_t - 1)}, \tag{A.1}$$

where $C_t(j)$ is the consumption of the good j. Maximization with respect to $C_t(j)$, for a given total expenditure, leads to a set of demand functions of the type

$$C_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon_t} C_t, \tag{A.2}$$

where $P_t(j)$ is the price of the good j. We let

$$\epsilon_t = \epsilon \exp \frac{1-\epsilon}{\epsilon} \mu_t, \quad \epsilon > 1, \tag{A.3}$$

where μ_t is an independent and identically distributed (i.i.d.) normal shock. The appropriate price deflator for the consumption basket is

$$P_t = \left(\int_0^1 P_t(j)^{1-\epsilon_t}\, dj\right)^{1/(1-\epsilon_t)}. \tag{A.4}$$

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Conditional on the optimal consumer behavior, $P_t C_t = [\int_0^1 P_t(j) C_t(j)\, dj]$. The representative household chooses sequences for consumption, savings, and leisure to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\chi_t \frac{(C_t - hC_{t-1})^{1-\sigma_c}}{1-\sigma_c} - \frac{N_t^{1+\sigma_n}}{1+\sigma_n} \right], \tag{A.5}$$

where χ_t is an exogenous demand shifter. Household maximization is subject to the sequence of budget constraints

$$P_t C_t + b_t B_t = B_{t-1} + W_t N_t. \tag{A.6}$$

Thus, the household holds its financial wealth in the form of one period bonds B_t with price b_t; W_t is the nominal wage and N_t is hours worked. The first order conditions of the problem are

$$0 = \chi_t (C_t - hC_{t-1})^{-\sigma_c} - \mathcal{L}_t, \tag{A.7}$$

$$0 = -N_t^{\sigma_n} + \mathcal{L}_t \frac{W_t}{P_t}, \tag{A.8}$$

$$1 = E_t \left[\beta \frac{\mathcal{L}_{t+1}}{\mathcal{L}_t} \frac{P_t}{P_{t+1}} R_t \right] = E_t \left[\beta \frac{\mathcal{L}_{t+1}}{\mathcal{L}_t} \frac{R_t}{\Pi_{t+1}} \right], \tag{A.9}$$

where $\mathcal{L}_t$ is the Lagrangian multiplier associated to the budget constraint and R_t is the gross nominal rate of return on bonds ($R_t = 1 + r_t = 1/b_t$). In the nonstochastic steady states,

$$w = W/P = N^{\sigma_n} (C - hC)^{\sigma_c},$$

$$1 = \beta R / \Pi.$$

A.2 *Firms*

There is a continuum of firms, indexed by $j \in [0, 1]$, that produce a differentiated good. They face the same technology

$$Y_t(j) = Z_t N_t(j)^{1-\alpha}, \tag{A.10}$$

where Z_t is an exogenous technology process. Firms pay a nominal wage W_t for every hour worked to the household. Following Calvo (1983), each firm may reset its price with probability $1 - \zeta_p$ in any given period, independently of time elapsed since last adjustment. Thus, a fraction $(1 - \zeta_p)$ chooses the price that maximizes nominal profits subject to a demand schedule, that is,

$$\max_{P_t(j)} \mathrm{Pr}_t = \max P_t(j) Y_t(j) - \mathrm{TC}_t(j) = \max (P_t(j) - \mathrm{MC}_t(j)) Y_t(j)$$

subject to $Y_t(j) = (\frac{P_t(j)}{P_t})^{-\epsilon_t} Y_t$. The first order conditions imply that

$$\left(P_t(j) - \frac{\epsilon_t}{\epsilon_t - 1} \mathrm{MC}_t(j) \right) Y_t(j) = 0.$$

Thus, the optimal price exceeds the marginal cost since the elasticity of goods variety exceeds 1.

For the fraction of firms ζ_p that cannot reoptimize prices, we assume

$$P_t(j) = P_{t-1}(j).$$

Let $\widetilde{V}_t$ be the value of a firm allowed to change prices at time t and let $V_t(P_{t-1}(i))$ be the value of a firm not allowed to change prices. Since the problem is identical for all firms of one type, they will choose the same optimal price. The value of a firm allowed to change the price is

$$\widetilde{V}_t = \max_{\widetilde{P}_t}\left[\Pr_t(\widetilde{P}_t) + \beta E_t \frac{Q_{t+1}}{Q_t}((1-\zeta_p)\widetilde{V}_{t+1} + \zeta_p V_{t+1}(\widetilde{P}_t))\right],$$

where $\frac{Q_{t+k}}{Q_t} = \frac{\mathcal{L}_{t+1}/P_{t+1}}{\mathcal{L}_t/P_t}$ is the stochastic discount factor. The value of the firm not allowed to change prices is

$$V_t(P_{t-1}) = \Pr_t(P_{t-1}) + \beta E_t \frac{Q_{t+1}}{Q_t}((1-\zeta_p)\widetilde{V}_{t+1} + \zeta_p V_{t+1}(P_{t-1})).$$

From the first order condition and the envelope theorem, we have

$$\begin{aligned} 0 &= \Pr'_t(\widetilde{P}_t) + \beta E_t \frac{Q_{t+1}}{Q_t}\zeta_p V'_{t+1}(\widetilde{P}_t), \\ V'_t(P_{t-1}) &= \Pr'_t(P_{t-1}) + \beta E_t \frac{Q_{t+1}}{Q_t}\zeta_p V'_{t+1}(P_{t-1}). \end{aligned} \tag{A.11}$$

Moving the latter equation forward, assuming $\widetilde{P}_t = P_t$, and iterating forword, we have

$$\begin{aligned} V'_{t+1}(\widetilde{P}_t) &= \Pr'_{t+1}(\widetilde{P}_t) \\ &\quad + E_{t+1}\frac{Q_{t+2}}{Q_{t+1}}\beta\zeta_p\left(\Pr'_{t+2}(\widetilde{P}_t) + E_{t+3}\frac{Q_{t+3}}{Q_{t+2}}\beta\zeta_p(\Pr'_{t+3}(\widetilde{P}_t) + \cdots)\right). \end{aligned}$$

Multiplying by $\frac{Q_{t+1}}{Q_t}\beta\xi_p$ and taking expectations conditional on time t information, we get

$$\begin{aligned} E_t\left(\frac{Q_{t+1}}{Q_t}\beta\zeta_p V'_{t+1}(\widetilde{P}_t)\right) &= E_t\left(\frac{Q_{t+1}}{Q_t}\beta\zeta_p \Pr'_{t+1}(\widetilde{P}_t) + \frac{Q_{t+2}}{Q_t}(\beta\zeta_p)^2 \Pr'_{t+2}(\widetilde{P}_t)\right. \\ &\qquad \left. + \frac{Q_{t+3}}{Q_t}(\beta\zeta_p)^3 \Pr'_{t+3}(\widetilde{P}_t) + \cdots\right) \\ &= E_t\left(\sum_{k=1}^{\infty}\frac{Q_{t+k}}{Q_t}(\beta\zeta_p)^k \Pr'_{t+k}(\widetilde{P}_t)\right). \end{aligned}$$

Substituting the latter into (A.11), we obtain

$$E_t\left(\sum_{k=0}^{\infty}\frac{Q_{t+k}}{Q_t}(\beta\zeta_p)^k \Pr'_{t+k}(\widetilde{P}_t)\right) = 0.$$

Substituting the first order condition for profit maximization, we get

$$E_t\left(\sum_{k=0}^{\infty}\frac{Q_{t+k}}{Q_t}(\beta\zeta_p)^k\left(1-\epsilon_{t+k}+\frac{\mathrm{MC}_{t+k}(i)}{\widetilde{P}_t}\epsilon_{t+k}\right)Y_{t+k}(j)\right)=0.$$

Cost minimization implies that the marginal cost is equal to the average cost, so

$$\begin{aligned}\mathrm{MC}_t(j)&=\mathrm{TC}_t(j)/Y_t(j)=\frac{W_tN_t(i)}{Y_t(j)}=\frac{W_t}{Y_t(j)}\left(\frac{Y_t(j)}{Z_t}\right)^{1/(1-\alpha)}\\&=W_tY_t(j)^{\alpha/(1-\alpha)}Z_t^{-1/(1-\alpha)}.\end{aligned}\tag{A.12}$$

Combining the marginal cost equation and the demand schedule, we get

$$\begin{aligned}\mathrm{MC}_t(i)&=W_tY_t(i)^{\alpha/(1-\alpha)}Z_t^{-1(1-\alpha)}\\&=W_t\left(\left(\frac{P_t(j)}{P_t}\right)^{-\epsilon_t}Y_t\right)^{\alpha/(1-\alpha)}Z_t^{-1/(1-\alpha)}\\&=W_tY_t^{\alpha/(1-\alpha)}Z_t^{-1/(1-\alpha)}\left(\frac{P_t(j)}{P_t}\right)^{-\alpha\epsilon_t/(1-\alpha)}=\mathrm{MC}_t\left(\frac{P_t(j)}{P_t}\right)^{-\alpha\epsilon_t/(1-\alpha)}.\end{aligned}\tag{A.13}$$

Thus the first order condition associated to the firm program is

$$\begin{aligned}&E_t\left(\sum_{k=0}^{\infty}\frac{Q_{t+k}}{Q_t}(\beta\zeta_p)^k\right.\\&\left.\times\left[1-\epsilon_{t+k}+\mathrm{MC}^r_{t+k}\,\epsilon_{t+k}\left(\frac{\widetilde{P}_t}{P_{t+k}}\right)^{\alpha-1-\alpha\epsilon_{t+k}/(1-\alpha)}\right]Y_{t+k}(j)\right)=0,\end{aligned}$$

where $\mathrm{MC}^r_{t+k}=\frac{\mathrm{MC}_{t+k}}{P_{t+k}}$ is the real (aggregate) marginal cost. In the nonstochastic steady state, the latter equation is verified if and only if the term inside the square brackets is zero; thus

$$1-\epsilon+\mathrm{MC}^r\;\epsilon\left(\frac{\widetilde{P}}{P}\right)^{(\alpha-1-\alpha\epsilon)/(1-\alpha)}=0.$$

Recall that the price deflator is $P_t=(\int_0^1 P_t(j)^{1-\epsilon_t}\,dj)^{1/(1-\epsilon_t)}$. The law of motion of prices is

$$1=\left(\zeta_p\left(\frac{P_{t-1}}{P_t}\right)^{1-\epsilon_t}+(1-\zeta_p)\left(\frac{\widetilde{P}_t}{P_t}\right)^{1-\epsilon_t}\right)^{1/(1-\epsilon_t)}.$$

In the steady state,

$$1=\left(\zeta_p\left(\frac{P}{P}\right)^{1-\epsilon}+(1-\zeta_p)\left(\frac{\widetilde{P}}{P}\right)^{1-\epsilon}\right)^{1/(1-\epsilon)}=\zeta_p+(1-\zeta_p)\left(\frac{\widetilde{P}}{P}\right)^{1-\epsilon}.$$

Thus, $\widetilde{P}=P$ and the real marginal cost in the steady state is $\mathrm{MC}^r=\frac{\epsilon-1}{\epsilon}$.

A.3 *Market clearing and aggregation*

Market clearing in the goods market requires $Y_t(j) = C_t(j)$. Letting the aggregate output be $Y_t \equiv (\int_0^1 Y_t(j)^{(\epsilon_t - 1)/\epsilon_t}\, dj)^{\epsilon_t/(\epsilon_t - 1)}$, we have $C_t = Y_t$. In the labor market, we have that

$$N_t = \int_0^1 N_t(j)\, dj = \int_0^1 \left(\frac{Y_t(j)}{Z_t}\right)^{1/(1-\alpha)} dj = \left(\frac{Y_t}{Z_t}\right)^{1/(1-\alpha)} \int_0^1 \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon_t/(1-\alpha)} dj.$$

Similarly, the aggregate real marginal cost is

$$\begin{aligned}
\mathrm{MC}_t^r &= \int_0^1 \mathrm{MC}_t^r(j)\, dj = \int_0^1 \frac{W_t/P_t N_t(j)}{Y_t(j)}\, dj \\
&= \frac{W_t}{P_t} \int_0^1 \frac{1}{Y_t(j)} \left(\frac{Y_t(j)}{Z_t}\right)^{1/(1-\alpha)} dj = \frac{W_t}{P_t} \left(\frac{1}{Z_t}\right)^{1/(1-\alpha)} \int_0^1 Y_t(j)^{\alpha/(1-\alpha)}\, dj \\
&= \frac{W_t}{P_t} \left(\frac{1}{Z_t}\right)^{1/(1-\alpha)} Y_t^{\alpha/(1-\alpha)} \int_0^1 \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon_t \alpha/(1-\alpha)} dj.
\end{aligned}$$

To sum up, the main equations of the model are

$$\begin{aligned}
&0 = \chi_t (C_t - hC_{t-1})^{-\sigma_c} - \mathcal{L}_t = \chi_t (C_t - hC_{t-1})^{-\sigma_c} - P_t Q_t, \\
&0 = N_t^{\sigma_n} - \mathcal{L}_t \frac{W_t}{P_t}, \\
&1 = E_t\left[\beta \frac{\mathcal{L}_{t+1}}{\mathcal{L}_t} \frac{R_t}{\Pi_{t+1}}\right], \\
&0 = E_t\left(\sum_{k=0}^{\infty} \frac{Q_{t+k}}{Q_t} (\beta \zeta_p)^k \right. \\
&\qquad \left. \times \left[1 - \epsilon_{t+k} + \mathrm{MC}_{t+k}^r\, \epsilon_{t+k} \left(\frac{\widetilde{P}_t}{P_{t+k}}\right)^{(\alpha - 1 - \alpha\epsilon_{t+k})/(1-\alpha)}\right] Y_{t+k}(j)\right), \\
&1 = \left(\zeta_p \left(\frac{P_{t-1}}{P_t}\right)^{1-\epsilon_t} + (1 - \zeta_p)\left(\frac{\widetilde{P}_t}{P_t}\right)^{1-\epsilon_t}\right)^{1/(1-\epsilon_t)}, \\
&Y_t = C_t, \\
&N_t = \left(\frac{Y_t}{Z_t}\right)^{1/(1-\alpha)} \int_0^1 \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon_t/(1-\alpha)} dj, \\
&\mathrm{MC}_t^r = \frac{W_t}{P_t} \left(\frac{1}{Z_t}\right)^{1/(1-\alpha)} Y_t^{\alpha/(1-\alpha)} \int_0^1 \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon_t \alpha/(1-\alpha)} dj.
\end{aligned}$$

We now derive the log-linearized conditions when either the technology process or the preference process has a nonstationary component.

A.4 *Nonstationary technology shock*

Assume that the preference shock is $\ln \chi_t = \rho_\chi \ln \chi_{t-1} + \epsilon_t^\chi$, where $\epsilon_t^\chi \sim N(0, \sigma_\chi^2)$, and that the technology process has two components, an autoregressive and a stochastic time trend, that is,

$$
\begin{aligned}
&Z_t = Z_t^c Z_t^T, \\
&\ln Z_t^T = bt + e_t^{Z,T}, \\
&\ln Z_t^c = \rho_z \ln Z_{t-1}^c + e_t^{Z,c}.
\end{aligned}
$$

The equilibrium conditions need to be rescaled by Z_t^T. Let $\widehat{Y}_t = \frac{Y_t}{Z_t^T}$, $\widehat{C}_t = \frac{C_t}{Z_t^T}$, $\widehat{W}_t = \frac{W_t}{Z_t^T}$, and $\widehat{\mathcal{L}}_t = \mathcal{L}_t (Z_t^T)^{\sigma_c}$, $\widehat{Q}_{t+k} = \widehat{\mathcal{L}}_{t+k} P_{t+k}$. Then

$$
\begin{aligned}
&N_t = \left(\frac{\widehat{Y}_t}{Z_t^c}\right)^{1/(1-\alpha)} \int_0^1 \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon_t/(1-\alpha)} dj, \\
&\mathrm{MC}_t^r = \frac{W_t}{P_t}\left(\frac{1}{Z_t}\right)^{1/(1-\alpha)} Y_t^{\alpha/(1-\alpha)} \int_0^1 \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon_t \alpha/(1-\alpha)} dj \\
&\quad = \frac{W_t}{P_t}\left(\frac{1}{Z_t^c}\right)^{1/(1-\alpha)} \left(\frac{1}{Z_t^T}\right)^{(1-\alpha+\alpha)/(1-\alpha)} Y_t^{\alpha/(1-\alpha)} \int_0^1 \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon_t \alpha/(1-\alpha)} dj \\
&\quad = \frac{\widehat{W}_t}{P_t}\left(\frac{1}{Z_t^c}\right)^{1/(1-\alpha)} \widehat{Y}_t^{\alpha/(1-\alpha)} \int_0^1 \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon_t \alpha/(1-\alpha)} dj, \\
&\widehat{C}_t = \widehat{Y}_t, \\
&\mathcal{L}_t (1/Z_t^T)^{-\sigma_c} = \chi_t \left(\widehat{C}_t - h\widehat{C}_{t-1} \frac{Z_{t-1}^T}{Z_t^T}\right)^{-\sigma_c}, \\
&\widehat{\mathcal{L}}_t = \chi_t (\widehat{C}_t - h\widehat{C}_{t-1} \exp\{-b + e_{t-1}^{Z,T} - e_t^{Z,T}\})^{-\sigma_c}, \\
&\widehat{N}_t^{-\sigma_n} = -\widehat{\mathcal{L}}_t \frac{\widehat{W}_t}{P_t},
\end{aligned}
$$

where $\widehat{N}_t = N_t/(Z_t^T)^{(\sigma_c - 1)/\sigma_n}$. Thus, if $\sigma_c = 1$, hours worked is stationary and consistency is ensured. The Euler equation becomes

$$
1 = E_t\left[\beta \frac{\widehat{\mathcal{L}}_{t+1}}{\widehat{\mathcal{L}}_t} \exp\{-b - e_{t+1}^{Z,T} + e_t^{Z,T}\} \frac{R_t}{\Pi_{t+1}}\right].
$$

The firm optimal condition when $\sigma_c = 1$ is

$$
\begin{aligned}
0 = E_t\Bigg(&\sum_{k=0}^{\infty} \frac{\widehat{Q}_{t+k}}{\widetilde{Q}_t} (\beta \zeta_p)^k \\
&\times \left[1 - \epsilon_{t+k} + \mathrm{MC}_{t+k}^r \, \epsilon_{t+k} \left(\frac{\widehat{P}_t}{P_{t+k}}\right)^{(\alpha - 1 - \alpha\epsilon_{t+k})/(1-\alpha)}\right] \widehat{Y}_{t+k}(j)\Bigg).
\end{aligned}
$$

Log linearization of the equilibrium conditions leads to

$$\lambda_t = \chi_t - \frac{1}{1-\overline{h}}(y_t - \overline{h}y_{t-1} - \overline{h}e^{Z,T}_{t-1} + \overline{h}e^{Z,T}_t),$$

$$w_t = \sigma_n n_t - \lambda_t,$$

$$y_t = z_t + (1-\alpha)n_t,$$

$$mc_t = \omega_t + n_t - y_t,$$

$$r_t = \rho_r r_{t-1} + (1-\rho_r)(\rho_y y_t + \rho_\pi \pi_t) + v_t,$$

$$\lambda_t = E_t[\lambda_{t+1} + r_t - \pi_{t+1} - e^{z,T}_{t+1} + e^{z,T}_t],$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa_p(\mu_t + mc_t),$$

where $\kappa_p = \frac{1-\alpha}{1-\alpha+\alpha\epsilon}\frac{(1-\beta\zeta_p)(1-\zeta_p)}{\zeta_p}$, $\overline{h} = e^{-b}h$, and variables in small letters are rescaled variables in log deviation from the steady state. Thus, in log deviations from the steady state,

$$\ln Y_t = bt + e^{Z,T}_t + y_t,$$

$$\ln W_t = bt + e^{Z,T}_t + w_t,$$

$$\ln \Pi_t = \pi_t,$$

$$\ln R_t = r_t.$$

A.5 *Nonstationary preference shock*

Assume that the technology shock is $\ln z_t = \rho_z \ln z_{t-1} + \epsilon^z_t$, where $\epsilon^z_t \sim N(0, \sigma^2_z)$, and that the preference process is

$$\chi_t = (\chi^T_t)^{1+\sigma_n}\chi^c_t,$$

$$\ln \chi^T_t = \ln \chi^T_{t-1} + e^{T,\chi}_t,$$

$$\ln \chi^c_t = \rho_\chi \ln \chi^c_{t-1} + e^{c,\chi}_t,$$

where $e^{j,\chi}_t \sim N(0, \sigma^2_{j,\chi})$ with $j = T, c$. Assume further that $\sigma_c = 1$ and $\alpha = 0$. Define $\widehat{C}_t = C_t/\chi^T_t$, $\widehat{Y}_t = Y_t/\chi^T_t$, $\widehat{N}_t = N_t/\chi^T_t$, $\widehat{\mathcal{L}}_t = \mathcal{L}_t(\chi^T_t)^{-\sigma_n}$, and $\widehat{Q}_{t+k} = \widehat{\mathcal{L}}_{t+k}P_{t+k}$. The equilibrium conditions become

$$\widehat{\mathcal{L}}_t = \frac{\chi^c_t}{\widehat{C}_t - h\widehat{C}_{t-1}\exp(-e^{T,\chi}_t)},$$

$$0 = \widehat{N}^{\sigma_n}_t + \widehat{\mathcal{L}}_t\frac{W_t}{P_t},$$

$$1 = \beta E_t\left[\frac{\widehat{\mathcal{L}}_{t+1}}{\widehat{\mathcal{L}}_t}R_t\frac{P_t}{P_{t+1}}\exp(\sigma_n\epsilon^{T,\chi}_{t+1})\right],$$

$$
\begin{aligned}
\widehat{N}_t &= \frac{\widehat{Y}_t}{Z_t}\int_0^1 \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon_t} dj,\\
\mathrm{MC}_t^r &= \frac{W_t}{P_t}\frac{1}{Z_t},\\
0 &= E_t\sum_{k=0}^{\infty}\frac{\widehat{Q}_{t+k}}{\widehat{Q}_t}\exp\left[(1+\sigma_n)\sum_{j=0}^{k-1}\epsilon_{t+k-1}^{\chi,P}\right]\\
&\quad\times(\beta\zeta_p)^k\left[1-\epsilon_{t+k}+\mathrm{MC}_{t+k}^r\,\epsilon_{t+k}\left(\frac{\widetilde{P}_t}{P_{t+k}}\right)^{(\alpha-1-\alpha\epsilon_{t+k})/(1-\alpha)}\right]\widehat{Y}_{t+k}(j).
\end{aligned}
$$

Log linearization leads to

$$
\begin{aligned}
\lambda_t &= \chi_t^c - \frac{1}{1-h}(y_t - hy_{t-1} + he_t^{T,\chi}),\\
w_t &= \sigma_n n_t - \lambda_t,\\
y_t &= z_t + n_t,\\
mc_t &= \omega_t + n_t - y_t,\\
r_t &= \rho_r r_{t-1} + (1-\rho_r)(\rho_y y_t + \rho_\pi \pi_t) + v_t,\\
\lambda_t &= E_t[\lambda_{t+1} + r_t - \pi_{t+1} + \sigma_n e_{t+1}^{T,\chi}],\\
\pi_t &= \beta E_t \pi_{t+1} + \kappa_p(\mu_t + mc_t),
\end{aligned}
$$

where variables in small letters are rescaled variables in log deviation from the steady state. Thus, in log deviations from the steady state,

$$
\begin{aligned}
\ln Y_t &= \chi_t^T + y_t,\\
\ln W_t &= w_t,\\
\ln \Pi_t &= \pi_t,\\
\ln R_t &= r_t.
\end{aligned}
$$

Appendix B: Supplementary tables for simulated data

This appendix reports the estimation results mentioned in the paper for alternative specifications of the DGP of the noncyclical component, for alternative combinations of filtered and unfiltered observables, and for different sample sizes.

Table A.1. Parameter estimates using different filters; all variables filtered; DGP1.

		Filter			
	True	LT Median (s.e.)	HP Median (s.e.)	FOD Median (s.e.)	BP Median (s.e.)
σ_c	1.0	1.13 (0.07)	1.15 (0.08)	1.07 (0.04)	1.07 (0.07)
σ_n	0.7	1.34 (0.06)	1.31 (0.06)	1.32 (0.05)	1.38 (0.06)
h	0.7	0.59 (0.03)	0.58 (0.03)	0.59 (0.02)	0.64 (0.02)
α	0.4	0.14 (0.02)	0.15 (0.02)	0.13 (0.01)	0.21 (0.02)
ϵ	7.0	3.85 (0.13)	4.51 (0.16)	4.19 (0.13)	3.81 (0.14)
ρ_r	0.2	0.74 (0.03)	0.73 (0.03)	0.67 (0.02)	0.68 (0.03)
ρ_π	1.3	1.45 (0.07)	1.53 (0.06)	1.59 (0.05)	1.54 (0.06)
ρ_y	0.05	0.48 (0.05)	0.46 (0.05)	−0.01 (0.00)	0.06 (0.02)
ζ_p	0.8	0.87 (0.03)	0.87 (0.03)	0.89 (0.03)	0.88 (0.03)
ρ_χ	0.5	0.74 (0.04)	0.76 (0.04)	0.42 (0.02)	0.99 (0.03)
ρ_z	0.8	0.40 (0.04)	0.46 (0.05)	0.99 (0.03)	0.57 (0.03)
σ_χ	1.12	0.19 (0.03)	0.19 (0.03)	0.16 (0.02)	0.07(0.01)
$\sigma_{z,c}$	0.51	0.07 (0.01)	0.07 (0.01)	0.15 (0.02)	0.07 (0.01)
σ_{mp}	0.12	0.10 (0.01)	0.09 (0.01)	0.11 (0.01)	0.07 (0.01)
σ_μ	20.64	1.78 (0.33)	1.50 (0.20)	6.28 (0.25)	0.60 (0.08)

Table A.2. Parameter estimates using different filters; real variables filtered; DGP1.

	Filter			
	LT Median (s.e.)	HP Median (s.e.)	FOD Median (s.e.)	BP Median (s.e.)
σ_c	1.14 (0.08)	1.15 (0.09)	1.27 (0.06)	1.21 (0.08)
σ_n	1.36 (0.07)	1.39 (0.08)	2.52 (0.11)	1.74 (0.11)
h	0.60 (0.03)	0.61 (0.03)	0.53 (0.03)	0.66 (0.03)
α	0.15 (0.02)	0.14 (0.02)	0.35 (0.03)	0.15 (0.03)
ϵ	3.98 (0.13)	3.37 (0.12)	4.10 (0.14)	4.19 (0.17)
ρ_r	0.75 (0.03)	0.75 (0.03)	0.71 (0.02)	0.66 (0.03)
ρ_π	1.66 (0.09)	1.66 (0.10)	1.61 (0.06)	1.45 (0.08)
ρ_y	0.49 (0.05)	0.58 (0.07)	−0.01 (0.00)	0.59 (0.06)
ζ_p	0.87 (0.03)	0.87 (0.03)	0.85 (0.03)	0.83 (0.03)
ρ_χ	0.73 (0.06)	0.78 (0.04)	0.30 (0.02)	0.82 (0.03)
ρ_z	0.45 (0.05)	0.39 (0.04)	0.99 (0.03)	0.24 (0.04)
σ_χ	0.19 (0.03)	0.23 (0.04)	0.83 (0.13)	0.48 (0.07)
$\sigma_{z,c}$	0.07 (0.01)	0.09 (0.01)	0.14 (0.02)	0.15 (0.02)
σ_{mp}	0.10 (0.01)	0.10 (0.01)	0.09 (0.01)	0.10 (0.01)
σ_μ	2.07 (0.31)	1.85 (0.27)	10.67 (0.49)	0.65 (0.14)

Table A.3. Parameter estimates using different filters; all variables filtered; DGP1.[a]

	Filter			
	LT Median (s.e.)	HP Median (s.e.)	FOD Median (s.e.)	BP Median (s.e.)
σ_c	1.12 (0.05)	1.13 (0.05)	1.06 (0.04)	1.07 (0.05)
σ_n	1.33 (0.05)	1.34 (0.05)	1.35 (0.05)	1.34 (0.05)
h	0.60 (0.02)	0.60 (0.02)	0.61 (0.02)	0.63 (0.02)
α	0.13 (0.01)	0.14 (0.01)	0.13 (0.01)	0.18 (0.01)
ϵ	4.18 (0.13)	3.97 (0.13)	4.00 (0.13)	4.06 (0.13)
ρ_r	0.77 (0.03)	0.76 (0.03)	0.68 (0.02)	0.70 (0.02)
ρ_π	1.60 (0.05)	1.59 (0.09)	1.53 (0.05)	1.60 (0.06)
ρ_y	0.49 (0.03)	0.41 (0.04)	−0.01 (0.00)	0.08 (0.01)
ζ_p	0.88 (0.03)	0.88 (0.03)	0.89 (0.03)	0.87 (0.03)
ρ_χ	0.55 (0.06)	0.33 (0.02)	0.36 (0.03)	0.99 (0.03)
ρ_z	0.44 (0.04)	0.49 (0.03)	0.99 (0.03)	0.71 (0.03)
σ_χ	0.12 (0.02)	0.09 (0.01)	0.16 (0.01)	0.04 (0.00)
$\sigma_{z,c}$	0.04 (0.00)	0.04 (0.00)	0.11 (0.01)	0.04 (0.00)
σ_{mp}	0.07 (0.01)	0.06 (0.01)	0.08 (0.01)	0.04 (0.00)
σ_μ	2.31 (0.19)	2.10 (0.16)	7.20 (0.31)	0.59 (0.05)

[a]Sample size is $T = 300$.

Table A.4. Parameter estimates using different filters; all variables filtered; DGP1; model with measurement error.

	Filter			
	LT Median (s.e.)	HP Median (s.e.)	FOD Median (s.e.)	BP Median (s.e.)
σ_c	1.13 (0.07)	1.13 (0.08)	1.08 (0.04)	1.02 (0.07)
σ_n	1.34 (0.06)	1.32 (0.06)	1.30 (0.06)	1.38 (0.06)
h	0.59 (0.03)	0.58 (0.03)	0.58 (0.03)	0.65 (0.02)
α	0.13 (0.02)	0.14 (0.03)	0.13 (0.02)	0.19 (0.02)
ϵ	3.67 (0.14)	4.20 (0.14)	4.13 (0.13)	4.03 (0.13)
ρ_r	0.73 (0.03)	0.72 (0.03)	0.67 (0.02)	0.68 (0.03)
ρ_π	1.59 (0.12)	1.60 (0.10)	1.55 (0.05)	1.62 (0.06)
ρ_y	0.45 (0.04)	0.41 (0.05)	−0.01 (0.00)	0.06 (0.01)
ζ_p	0.88 (0.03)	0.87 (0.03)	0.89 (0.03)	0.88 (0.03)
ρ_χ	0.76 (0.04)	0.78 (0.04)	0.45 (0.02)	0.99 (0.03)
ρ_z	0.45 (0.05)	0.39 (0.06)	0.99 (0.03)	0.59 (0.04)
σ_χ	0.19 (0.03)	0.19 (0.03)	0.17 (0.02)	0.07 (0.01)
$\sigma_{z,c}$	0.07 (0.01)	0.07 (0.01)	0.15 (0.02)	0.07 (0.01)
σ_{mp}	0.10 (0.01)	0.09 (0.01)	0.11 (0.01)	0.07 (0.01)
σ_μ	1.87 (0.24)	1.44 (0.24)	6.32 (0.35)	0.58 (0.07)
σ_{me1}	0.61 (0.20)	0.68 (0.26)	0.51 (0.09)	0.70 (0.31)
σ_{me2}	0.64 (0.19)	0.62 (0.21)	0.75 (0.19)	0.58 (0.15)
σ_{me3}	0.68 (0.21)	0.66 (0.31)	0.89 (0.25)	0.56 (0.18)
σ_{me4}	0.56 (0.25)	0.68 (0.19)	0.64 (0.11)	0.68 (0.30)

TABLE A.5. Parameter estimates using different filters; real variables filtered; DGP2.

	Filter			
	LT Median (s.e.)	HP Median (s.e.)	FOD Median (s.e.)	BP Median (s.e.)
σ_c	3.90 (0.36)	4.71 (0.25)	3.23 (0.86)	5.22 (0.25)
σ_n	0.30 (0.05)	0.20 (0.02)	0.28 (0.03)	0.06 (0.03)
h	0.59 (0.03)	0.56 (0.03)	0.70 (0.02)	0.87 (0.03)
ϵ	4.20 (0.14)	4.00 (0.13)	4.10 (0.13)	4.02 (0.13)
ρ_r	0.30 (0.01)	0.29 (0.02)	0.56 (0.02)	0.16 (0.02)
ρ_π	1.75 (0.07)	1.67 (0.06)	1.56 (0.05)	1.48 (0.05)
ρ_y	−0.03 (0.01)	−0.08 (0.02)	0.03 (0.02)	−0.13 (0.01)
ζ_p	0.83 (0.03)	0.84 (0.03)	0.81 (0.03)	0.86 (0.03)
ρ_χ	0.62 (0.06)	0.39 (0.02)	0.48 (0.02)	0.79 (0.03)
ρ_z	0.72 (0.03)	0.67 (0.02)	0.48 (0.02)	0.38 (0.02)
$\sigma_{\chi,c}$	0.15 (0.03)	0.17 (0.02)	0.72 (0.38)	0.37 (0.07)
σ_z	0.15 (0.02)	0.20 (0.03)	0.38 (0.08)	5.15 (0.25)
σ_v	0.03 (0.00)	0.03 (0.00)	0.03 (0.00)	0.03 (0.00)
σ_μ	7.28 (0.49)	8.92 (0.46)	4.95 (0.23)	3.74 (0.33)

TABLE A.6. Posterior parameter estimates.[a]

	True	Factor 1 Median (s.e.)	Factor 2 Median (s.e.)
σ_c	1.00	0.87 (0.10)	1.72 (0.10)
σ_n	0.70	0.73 (0.06)	0.29 (0.09)
h	0.70	0.56 (0.10)	0.62 (0.03)
α	0.40	0.34 (0.04)	0.32 (0.03)
ϵ	7.00	6.29 (0.13)	6.45 (0.14)
ρ_r	0.20	0.67 (0.03)	0.80 (0.04)
ρ_π	1.30	1.61 (0.03)	1.51 (0.02)
ρ_y	0.05	0.40 (0.03)	0.31 (0.04)
ζ_p	0.80	0.85 (0.03)	0.85 (0.03)
ρ_χ	0.50	0.82 (0.06)	0.69 (0.08)
ρ_z	0.80	0.70 (0.03)	0.69 (0.02)
$\sigma_{\chi,c}$	1.10	0.22 (0.04)	0.21 (0.04)
σ_z	0.57	0.18 (0.03)	0.29 (0.08)
σ_v	0.12	0.13 (0.02)	0.10 (0.01)
σ_μ	20.64	6.51 (1.11)	6.07 (1.25)

[a]Factor 1 uses LT, HP, and FOD filtered data; Factor 2 uses HP ($\lambda = 1600$), HP ($\lambda = 6400$), and BP filtered data. The DGP features a technology shock with two components: a stationary AR(1) and a unit root. All variables are filtered prior to estimation. The sample size is $T = 150$.

APPENDIX C: SUPPLEMENTARY TABLES FOR ACTUAL DATA

This appendix reports the estimation results for all the parameters for the Ireland (2004) model presented in Section 5.

TABLE A.7. Structural parameter estimates for a model with money.

Parameter	Prior[a]	Basic Model Median (s.e.)	Ireland's Specification Median (s.e.)
ω_1	$\Gamma(1, 0.1)$	1.03 (0.02)	0.98 (0.01)
ω_2	$\Gamma(1, 0.1)$	0.44 (0.02)	0.03 (0.01)
ψ	$N(1.0, 0.1)$	1.02 (0.02)	0.98 (0.03)
ρ_r	$B(2, 6)$	0.59 (0.01)	0.59 (0.01)
ρ_π	$N(1.5, 0.2)$	1.51 (0.02)	1.40 (0.01)
ρ_y	$N(0.2, 0.2)$	0.44 (0.01)	0.45 (0.01)
ρ_m	$N(1.0, 0.2)$	0.48 (0.02)	0.04 (0.02)
γ_1	$\Gamma(10, 0.1)$	0.92 (0.02)	1.00 (0.01)
γ_2	$\Gamma(5, 0.1)$	0.51 (0.01)	0.51 (0.01)
ρ_a	$B(8, 8)$	0.72 (0.01)	0.67 (0.01)
ρ_e	$B(8, 8)$	0.77 (0.01)	0.79 (0.03)
ρ_z	$B(22, 8)$	0.74 (0.04)	0.88 (0.01)
σ_a	$\Gamma^{-1}(5, 20)$	0.74 (0.10)	1.30 (0.07)
σ_e	$\Gamma^{-1}(5, 20)$	0.81 (0.08)	2.02 (0.16)
σ_z	$\Gamma^{-1}(5, 20)$	0.18 (0.03)	0.46 (0.40)
σ_v	$\Gamma^{-1}(5, 20)$	0.37 (0.06)	0.52 (0.15)

[a] Γ is the gamma distribution; B is the beta distribution; N is the normal distribution.

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TABLE A.8. Additional parameter estimates for a model with money.

Parameter	Prior[a]	Basic Model Median (s.e.)	Ireland's Specification Median (s.e.)
ν_0^{po}	$N(0, 0.1)$	0.00 (0.00)	
ν_0^{fd}	$N(0, 0.1)$	−0.00 (0.00)	
ν_0^{hp1}	$N(0, 0.1)$	−0.00 (0.00)	
ν_0^{hp2}	$N(0, 0.1)$	−0.00 (0.00)	
ν_0^{cum}	$N(0, 0.1)$	0.00 (0.00)	
ν_0^{cd}	$N(0, 0.1)$	0.00 (0.00)	
ν_0^{uc}	$N(0, 0.1)$	0.00 (0.00)	
ν_0^{mbn}	$N(0, 0.1)$	0.00 (0.00)	
ν_1^{fd}	$N(1, 0.5)$	0.73 (0.01)	
ν_1^{hp1}	$N(1, 0.5)$	0.82 (0.02)	
ν_1^{hp2}	$N(1, 0.5)$	0.77 (0.01)	
ν_1^{cum}	$N(1, 0.5)$	0.77 (0.02)	
ν_1^{cd}	$N(1, 0.5)$	0.86 (0.02)	
ν_1^{uc}	$N(1, 0.5)$	0.70 (0.05)	
ν_1^{mbn}	$N(1, 0.5)$	0.78 (0.01)	
σ_y^{po}	$\Gamma^{-1}(10, 30)$	0.12 (0.01)	0.12 (0.01)
σ_m^{po}	$\Gamma^{-1}(10, 30)$	0.24 (0.03)	0.06 (0.01)
σ_π^{po}	$\Gamma^{-1}(10, 30)$	0.06 (0.01)	0.09 (0.01)
σ_r^{po}	$\Gamma^{-1}(10, 30)$	0.06 (0.01)	0.07 (0.01)
σ_y^{fd}	$\Gamma^{-1}(10, 30)$	0.03 (0.00)	
σ_w^{fd}	$\Gamma^{-1}(10, 30)$	0.04 (0.00)	
σ_π^{fd}	$\Gamma^{-1}(10, 30)$	0.03 (0.00)	
σ_r^{fd}	$\Gamma^{-1}(10, 30)$	0.03 (0.00)	
σ_y^{hp1}	$\Gamma^{-1}(10, 30)$	0.05 (0.01)	
σ_w^{hp1}	$\Gamma^{-1}(10, 30)$	0.05 (0.01)	
σ_π^{hp1}	$\Gamma^{-1}(10, 30)$	0.04 (0.00)	
σ_r^{hp1}	$\Gamma^{-1}(10, 30)$	0.04 (0.00)	
σ_y^{hp2}	$\Gamma^{-1}(10, 30)$	0.05 (0.00)	
σ_w^{hp2}	$\Gamma^{-1}(10, 30)$	0.05 (0.01)	
σ_π^{hp2}	$\Gamma^{-1}(10, 30)$	0.04 (0.00)	
σ_r^{hp2}	$\Gamma^{-1}(10, 30)$	0.04 (0.00)	
σ_y^{cum}	$\Gamma^{-1}(10, 30)$	0.04 (0.00)	
σ_w^{cum}	$\Gamma^{-1}(10, 30)$	0.07 (0.01)	
σ_π^{cum}	$\Gamma^{-1}(10, 30)$	0.04 (0.00)	
σ_r^{cum}	$\Gamma^{-1}(10, 30)$	0.03 (0.00)	
σ_y^{cd}	$\Gamma^{-1}(10, 30)$	0.12 (0.01)	
σ_w^{cd}	$\Gamma^{-1}(10, 30)$	0.25 (0.03)	
σ_π^{cd}	$\Gamma^{-1}(10, 30)$	0.06 (0.01)	
σ_r^{cd}	$\Gamma^{-1}(10, 30)$	0.06 (0.01)	
σ_y^{uc}	$\Gamma^{-1}(10, 30)$	0.03 (0.00)	
σ_w^{uc}	$\Gamma^{-1}(10, 30)$	0.04 (0.00)	
σ_π^{uc}	$\Gamma^{-1}(10, 30)$	0.04 (0.00)	
σ_r^{uc}	$\Gamma^{-1}(10, 30)$	0.03 (0.00)	
σ_y^{mbn}	$\Gamma^{-1}(10, 30)$	0.06 (0.01)	
σ_w^{mbn}	$\Gamma^{-1}(10, 30)$	0.07 (0.01)	
σ_π^{mbn}	$\Gamma^{-1}(10, 30)$	0.04 (0.00)	
σ_r^{mbn}	$\Gamma^{-1}(10, 30)$	0.04 (0.00)	

[a] Γ is the gamma distribution, N is the normal distribution.